Předmět **Advanced Analysis topics**
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Dear Martin,

Following our yesterday discussion, I prepared a list of topic for elements of advanced analysis for bachelors. Each topic approximately corresponds to a 2 hour lecture, plus-minus. I thought carefully and, in my opinion, Functional Analysis (metric and normed spaces, etc...) is not feasible in the format we discuss, in my opinion. These concepts are extremely difficult even for strong students, and really require a lot of examples, time and patience. That is, it really should be a separate course. That is why I only included below stuff on L^p spaces, which is probably the most important for applications.

All the best,
Ilya.

Complex Analysis topics

1. Functions of complex variable. Limit and continuity. C-differentiability. Cauchy-Riemann conditions. Holomorphic functions.

2. Integration on complex plane. Cauchy integral theorem (deduced immediately from Stokes theorem!). Cauchy integral formula. Residue formula and applications.

3. Applications of the Cauchy formula: C^\infty property; power series expansion (complex-analyticity); Weierstrass convergence theorem; maximum modulus principle; uniqueness theorem.

4. Isolated zeroes and singularities of holomorphic functions. Laurent series.

Measure and integral theory, Fourier analysis topics

1. Upper and lower Lebesgue measures of a bounded set. Measurable sets. Elementary properties of a measure.

2-3. Measurable functions and Lebesgue integral (via characteristic functions of measurable sets). Algebraic properties, absolute integrability, absolute continuity. Comparison with Riemann integral. Integral as measure. Lebesgue convergence theorem. Fubini theorem.

4. L^p spaces. Holder inequality. L^2 space, scalar product and its properties.

5. Orthonormal systems and Fourier series in L^2. Properties.

6. Fourier transform and its properties