Vznik řady matematických disciplín

Teorie čísel

Pierre de Fermat (1601 – 1665)

Vynikající klasické vzdělání: latina, řečtina, italština, španělština.

Matematika: nejpozději konec 20. let v Bordeaux,

Vietovy spisy

od 1636 - teorie čísel

Toulouse - Pierre de Carcavi (1600-1684)- 1636 přeložen do Paříže –

Marin Mersenne (1588-1648)

1636 - 1643 Malá a Velká Fermatova věta

Od 1670 - po Fermatově smrti - syn Samuel shromažďuje roztroušenou korespondenci. Vydává znovu Diofantovu *Aritmetiku* s otcovými poznámkami. K vydání připojena práce Jacquese de Billyho *Doctrinae Analyticae Inventum Novum*, psaná podle Fermatových dopisů - diofantické rovnice.

1679 - vydána práce *Varia Opera* (geometrie, algebra, diferenciální a integrální počet, dopisy)

Co vedlo Fermata k vybudování základů teorie čísel?

Motiv: vybudování aritmetiky jako nauky o celých číslech (na rozdíl od Diofanta)

prvočísla tvaru 4k+1 lze vyjádřit jako součet druhých mocnin, prvočísla tvaru 4k-1 nikoliv

Metoda nekonečného sestupu

Velká Fermatova věta

„Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem fane detexi. Hanc marginis exiguitasnon caperet.“

„Nelze rozdělit krychli na dvě krychle, bikvadrát na dva bikvadráty a obecně žádnou mocninu vyšší než dvě na dvě mocniny téhož stupně. Pro tuto skutečnost jsem nalezl podivuhodný důkaz, tento okraj je však příliš úzký.“

Definitivní vyřešení problematiky Velké Fermatovy věty:

Andrew Wiles (1953) 1993, 1995

2. oblast: Malá Fermatova věta

Věta: Nechť p je prvočíslo. Pak pro všechna přirozená a platí

$$a^p \equiv a \pmod p.$$

Je-li navíc (a,p) = 1, platí

$$a^{p-1} \equiv 1 \pmod p.$$

Symbol kongruence ovšem zavedl až Gauss (1777-1855)

Femat studoval *dokonalá čísla*

$$s(n) = n\,, \quad S(n) = 2n\,.$$

1638 René Descartes (1596 - 1650) dokázal

Věta: Nechť (a,b) = 1. Pak

$$s(ab) = s(a).s(b) + a.s(b) + b.s(a)\,.$$

Věta: Nechť (a,b) = 1. Pak

$$S(ab) = S(a).S(b).$$

Věta: Sudé n je dokonalé právě tehdy, když je tvaru

$$n = 2^{k-1}.\,(2^k-1)\,,$$

kde $k > 1$ a $2^k - 1$ je prvočíslo.

Dostatečnost podmínky znal už Eukleides, nutnost však dokázal až EULER (1707 - 1783).

ŘEKOVÉ: znali dokonalá čísla 6, 28, 496, 8128

$$6 = 1 + 2 + 3, \qquad 28 = 1 + 2 + 4 + 7 + 14$$

EUKLEIDES : Je-li M_n prvočíslo, je $2^{n-1} . M_n$ dokonalé.

EULER: Sudá dokonalá čísla jsou právě uvedeného tvaru.

1814 BARLOW : $2^{30}.M_{31}$ je největší dokonalé číslo, jaké kdy bylo objeveno.

PROBLÉM : Existuje liché dokonalé číslo?

Pokud ano, musí být větší než 10^{200}, musí mít alespoň 8 prvočíselných dělitelů, z nichž aspoň jeden musí být větší než 300 000; je-li menší než $10^{9\,118}$, musí být dělitelné 6. mocninou některého prvočísla, ...

Fermatova prvočísla

$F_m = 2^{2} + 1$ pro $m=0,1,2,\ldots$ jsou prvočísla

$F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65\,537$

Jak na to přišel?

Věta: **Je-li p přirozené a $q > 1$ liché, platí**

$$2^{pq} + 1 = (2^p + 1)(2^{p(q-1)} - 2^{p(q-2)} + \ldots - 2^p + 1)$$

Důsledek: **Je-li číslo $2^n + 1$ prvočíslo, musí být exponent n tvaru $k=2^m$.**

1732 L. EULER $F_5 = 2^{32} + 1 = 4\,294\,967\,297 =$
$= 641 \,.\, 6\,700\,417$

1880 F. LANDRY rozložil číslo

$$F_6 = 2^{64} + 1 = 274\,147 \times 67\,280\,421\,310\,721$$

Carl Friedrich GAUSS (1777 - 1855):

Věta: Pravidelný mnohoúhelník je eukleidovsky konstruovatelný právě tehdy, když počet jeho vrcholů je roven číslu

$$k = 2^i p_1 p_2 \dots p_j,$$

kde $p_1, p_2, \dots, p_j$ jsou navzájem různá Fermatova prvočísla.

Je konstruovatelný: $k = 3,4,5,6,8,10,12,15,16,17,\dots\dots$

Není konstruovatelný: $k = 7,9,11,13,14,\dots$

Známé konstrukce: $k = 17, 257, 65\,537$

Zatím známe $31 = 2^5 - 1$ eukleidovsky konstruovatelných mnohoúhelníků s lichým počtem vrcholů.

1897 Felix KLEIN (1849 - 1925) F_7 je složené, neurčil však žádného dělitele

1909 totéž pro F_8 J. C. Moreheard a A. E. Western

1970 $F_7 = (2^9 \,.\, 116\,503\,103\,764\,643 + 1)\,.$
$\,.\,(2^9 \,.\, 11\,141\,971\,095\,088\,142\,685 + 1)$

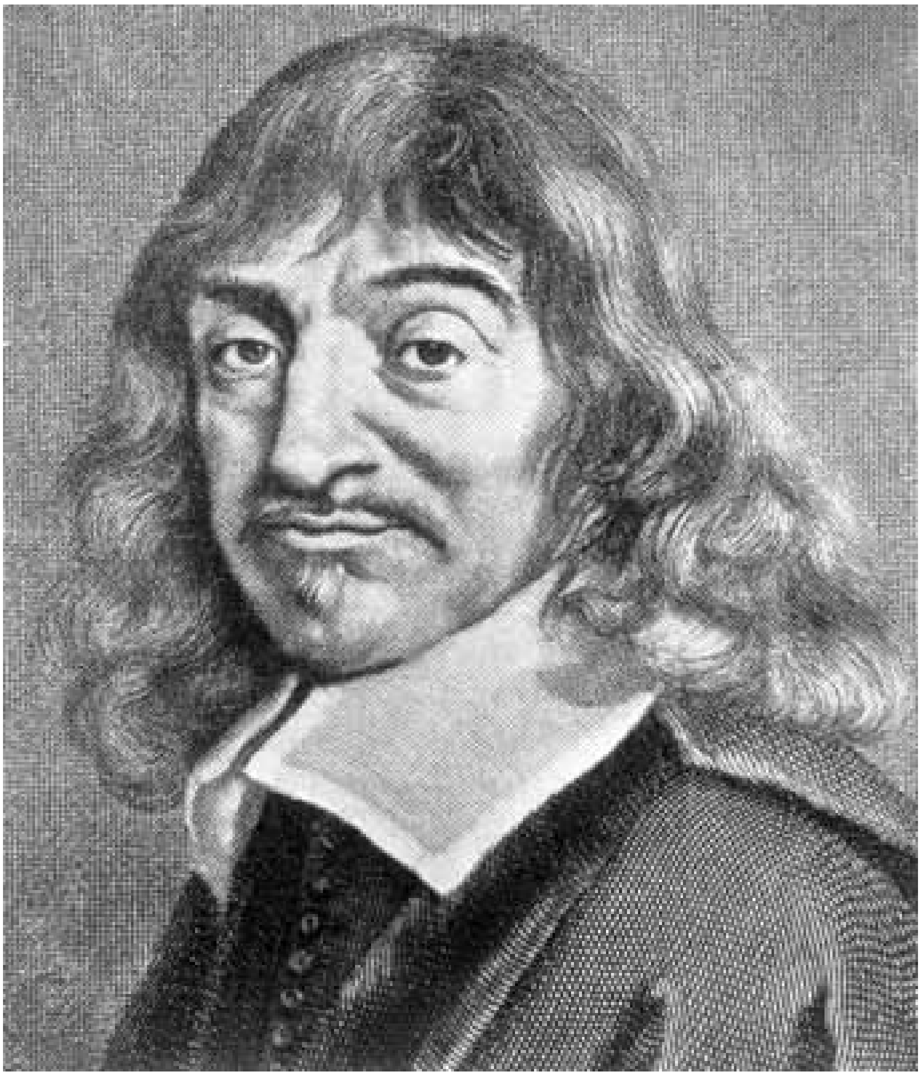
Zkoumejme prvočíselnost čísla F_m standardně - dělením všemi prvočísly menšími než F_m. Jak dlouho bychom ověřovali např. F_8?

Celá část F_8 má 39 cifer, takže před F_8 je cca

$$10^{38}/(38.\ln 10) = 10^{36}$$

prvočísel. Rok má cca $3{,}2 \cdot 10^7$ sekund, takže při miliardě dělení za sekundu bychom potřebovali cca $3 \cdot 10^{19}$ let. Stáří vesmíru je cca $15 \cdot 10^9$ let.

ANALYTICKÁ GEOMETRIE



René DESCARTES (1596 – 1650)

1637 Descartes ***Geometrie (Rozprava o metodě)***
FERMAT Ad locos planos et solidos isagoge
(Úvod do studia rovinných křivek a ploch)

TEORIE PRAVDĚPODOBNOSTI

Blaise PASCAL (1623 – 1662)

1654 - korespondence Fermata s Blaisem Pascalem

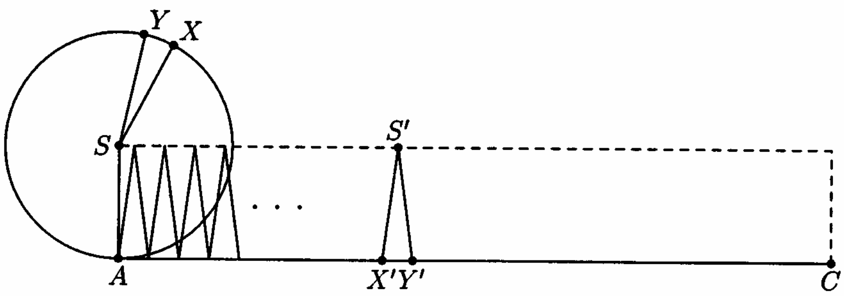
Christian HUYGENS (1629 – 1695)

1657 ***O počítání při hře v kostky čili o počítání při hazardních hrách***

Počátky kalkulu

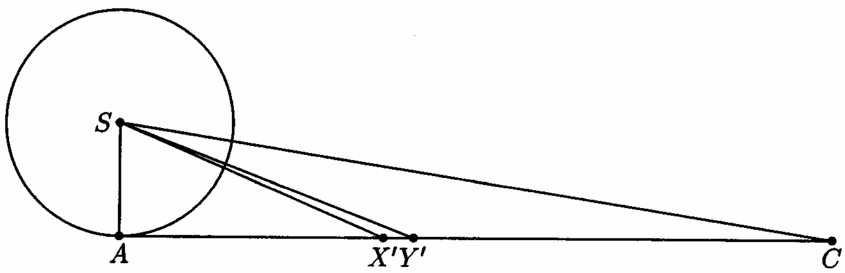
Johannes KEPLER (1571 – 1630)

1615 ***Nová stereometrie vinných sudů***



Obr. 3. Keplerův výpočet obsahu kruhu

Tyto trojúhelníky lze zaměnit jinými, se stejnými základnami a výškou, přičemž vrcholy všech trojúhelníků se posunou do středu kružnice S. Takto vzniklé trojúhelníky mají stejné obsahy jako původní trojúhelníky a dohromady vyplňují trojúhelník ACS.

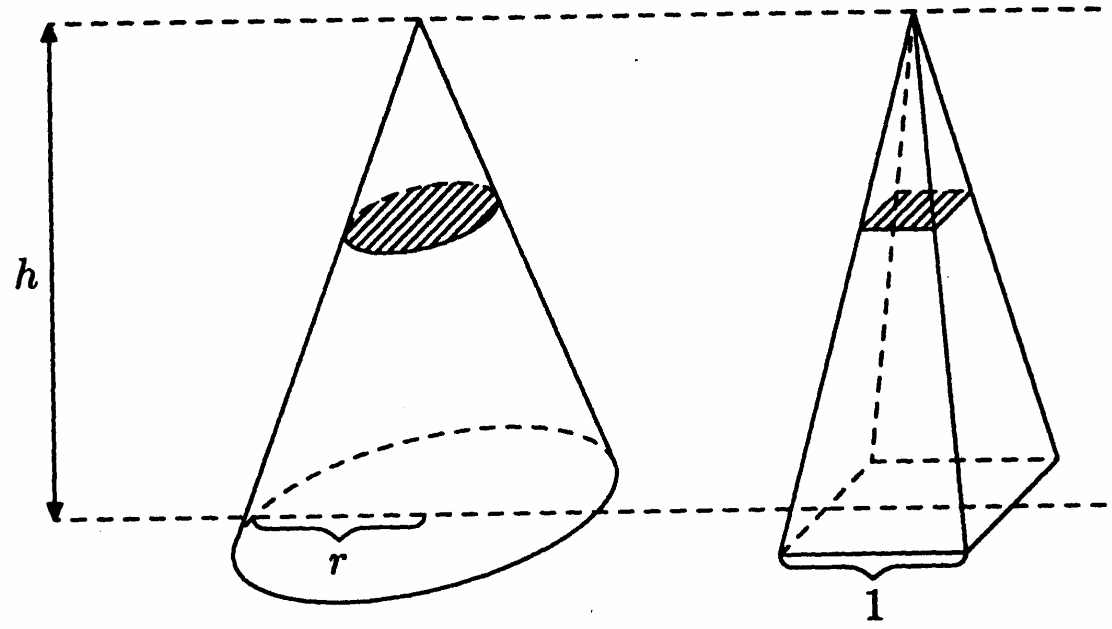


Obr. 4. Keplerův výpočet obsahu kruhu

Obsah kruhu je tedy roven obsahu

Bonaventuera CAVALIERI (1598 – 1647)

1535 Geometria indivisibilibus continuorum



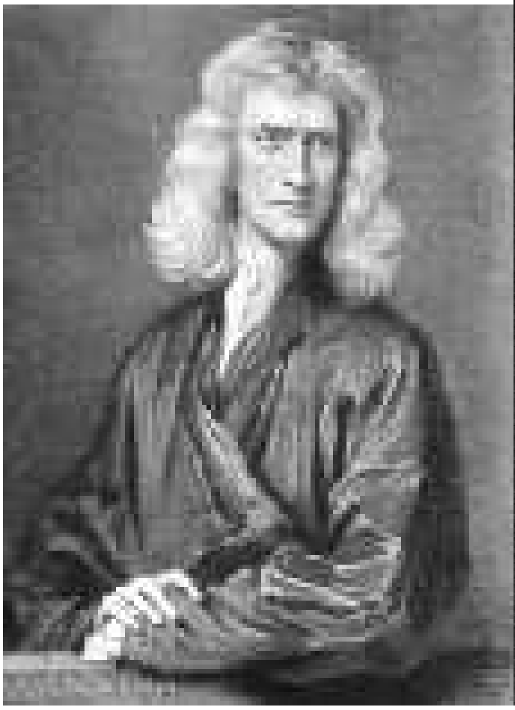
Obr. 5. Cavalieriho princip

Když dvě tělesa mají stejnou výšku a když řezy rovinami, které jsou rovnoběžné s jejich podstavami a mají od nich stejnou vzdálenost, jsou takové, že poměr jejich obsahů je vždy stejný, potom objemy těles mají týž poměr.

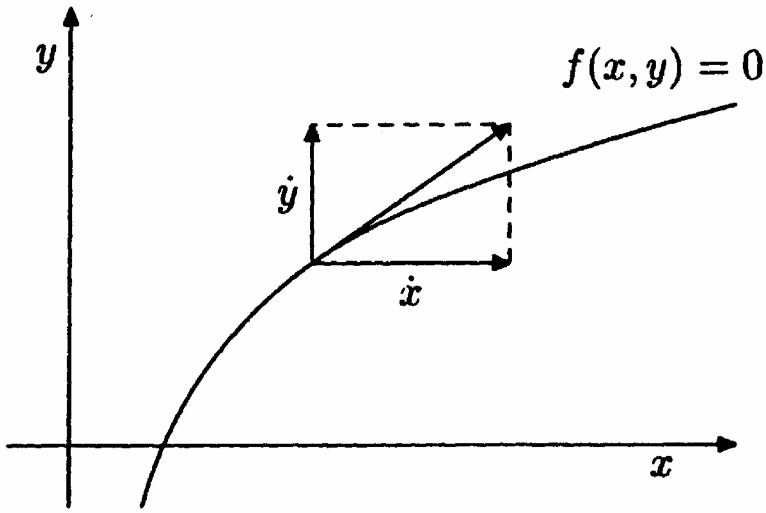
John WALLIS (1616 – 1703)

1655 *Arithmetica infinitorum*

Christiaan HUYGENS
Isaac BARROW (1630 – 1677)

[Isaac NEWTON](#) (1642 – 1727)



Obr. 10. Newtonova metoda fluxí

x, y **fluenty - popisují dráhu hmotného bodu**
x, y **fluxe - rychlosti, s nimiž se veličiny mění**
čas pomocný pojem, příklad nezávisle proměnné veličiny; jeho nekonečně malý přírůstek značí *o*
xo, yo **nekonečně malé přírůstky fluent (momenty fluxí)**

Dvě základní Newtonovy úlohy:

1. **Nalézt vztah mezi fluxemi *x* a *y*, je-li dán vztah mezi fluentami rovnicí *f*(x,y) = 0 .**
2. **Nalézt vztah mezi fluentami *x,y* , tj. nalézt funkci *f* vyhovující rovnici *f*(x,y) = 0 , je-li dán vztah mezi fluxemi.**

Postup:

1. **Mějme vztah f(x,y) = 0 mezi fluentami. V „následujícím“ okamžiku tedy platí**

f(x+xo,y+yo) = 0 . Tento vztah zjednodušíme, vydělíme „o“ a zanedbáme všechny členy s „o“, protože v porovnání s členy bez „o“ jsou „pouhé nic“.

Příklad: Mějme funkci $y = x^n$. Dosaďme do rovnice $y - x^n = 0$, dostaneme

$$(y - yo) - (x - xo)^n = 0.$$

Podle binomické věty, po dosazení $y = x^n$ a po vydělení členem „o“ dostaneme

$$\dot{y} - nx^{n-1}\left(\dot{x}\right) - \binom{n}{2} x^{n-2}\left(\dot{x}\right)^2 o - \ldots = 0 \quad ,$$

odkud

$$\frac{\dot{y}}{\dot{x}} = nx^{n-1} \quad ,$$

neboť členy obsahující „o“ jsou „pouhé nic“.

Gottfried Wilhelm LEIBNIZ (1646 – 1716)

Idea Pascalova *charakteristického trojúhelníka:*

Naskenovat obrázek:

Z podobnosti trojúhelníků dostáváme

$\frac{m}{x} = \frac{dy}{dx}$ neboli $mdx = ydy$.

Tuto situaci si představil v každém bodě a veličiny na obou stranách rovnosti sečetl. Tyto součty nekonečně mnoha nekonečně malých veličin označil symbolem ∫ .(Název „integrál“ zavedl Jacob Bernoulli.) Dostal tak

$$\int mdx = \int ydy \, .$$

Úpravou a přepsáním do tvaru určitého integrálu obdržel

$$\int_a^b y\frac{dy}{dx}dx = \int_{y(a)}^{y(b)} ydy = \frac{1}{2}\left[y^2\right]_{y(a)}^{y(b)} = \frac{1}{2}\left[y(b)^2 - y(a)^2\right].$$

Když chtěl například určit integrál funkce x^n , vedl úvahy tak, aby učil funkci y , pro kterou by bylo $y(dy/dx) = x^n$. Položil tedy

$$y(x) = \alpha x^k$$

a hledal odpovídající hodnoty α a k .
Po dosazení dostal $k = (n+1)/2$, $\alpha = (\sqrt{2})/(\sqrt{(n+1)})$.

Takto vyjádřenou funkci pak dosadil do

výše uvedeného vztahu pro integrál y obdržel známou formuli

$$\int_a^b x^n dx = \frac{1}{2}\left[y^2\right]_{y(a)}^{y(b)} = \frac{1}{2}\left[\frac{2}{n+1}b^{n+1} - \frac{2}{n+1}a^{n+1}\right] = \frac{1}{n+1}\left(b^{n+1} - a^{n+1}\right).$$

Leibniz kladl velký důraz na symboliku. Vytvářel ji tak, aby usnadňovala pochopení podstaty pojmů.

29. 10. 1675 píše:
Bude užitečné místo „součtu všech *l*" psát od nynějška $\int l$ (znak $\int$ je odvozen z prvního písmene slova summa)… vzniká nový druh počtu, nová početní operace, která odpovídá sčítání a násobení. Druhý druh počtu vzniká, když z výrazu $\int l = a$ získáme $l = a(y/d)$ (d je první písmeno slova differentia). Jako totiž operace $\int$ zvětšuje rozměr, tak jej d zmenšuje.

John WALLIS

Stál u ustavení *Londýnské královské společnosti*

1733 Voltaire *Listy o Angličanech*:
„…každý Angličan, který sám sebe prohlásí za milovníka matematiky a fyziky a projeví zájem stát

se lenem královské společnosti, je do ní okamžitě zvolen.“

1685 Wallis: ***Pojednání o algebře***
„1939 Cohen: .. „jedna z největších manipulací s fakty v historii vědy. … Wallis naznačuje, že všechny velké matematické objevy 17. století učinili Angličané a že například Descartes opisoval od Harriota.“

Slavný spor Johna Wallise a Thomase Hobbese (po uveřejnění Hobbsovy knihy o kvadratuře kruhu):

Wallis (v dopise Huyghensovi 1. 1. 1659):
„…je nutné, aby mu nějaký matematik ukázal, jak málo matematice, z níž čerpá svou troufalost, rozumí. A nesmíme se nchat odradit jeho nadutostí, v níž na nás bude, jak víme, plivat sliny.“

Hobbes: ***Šest lekcí profesorům matematiky, jedna pro profesora geometrie a zbývající pro profesora astronomie.*** **„…Tak kráčejte cestami svými, vy neslušní kněží, nelidští teologové, dedikátoři morálky, zavilí kolegové, vy dva ohavní Issacharové, nejzkaženější mstitelé a zrádci akademie.“**

1684 Leibniz zveřejnil první práci

Wallis byl dotčen tím, že Němci by se měli dostat před Anličany. 1695 píše Newtonovi: „Nejste tak

laskav ke své pověsti (a potažmo pověsti národa), jak byste mohl být, jestliže tak hodnotné věci necháváte u sebe ležet tak dlouho, až jiní na sebe strhnou slávu, která patří vám.“

1707 John Keill (Philosophical Transactions): „…Newtonovo prvenství existuje mimo jakýkoliv stín pochybnosti.“

Johann Bernoulli o Keillovi: „Newtonova opice“, „Newtonův patolízal“, najaté péro“, „jistý jedinec skotské rasy“.

1734 George BERKELEY: „duchové zemřelých veličin“. Jednou jsou nulové a pak zase nejsou, podle potřeby operací, s nimiž se provádějí.

1820 CAUCHY

Diophantus of Alexandria

Born: about 200
Died: about 284

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Diophantus, often known as the 'father of algebra', is best known for his *Arithmetica*, a work on the solution of algebraic equations and on the theory of numbers. However, essentially nothing is known of his life and there has been much debate regarding the date at which he lived.

There are a few limits which can be put on the dates of Diophantus's life. On the one hand Diophantus quotes the definition of a polygonal number from the work of Hypsicles so he must have written this later than 150 BC. On the other hand Theon of Alexandria, the father of Hypatia, quotes one of Diophantus's definitions so this means that Diophantus wrote no later than 350 AD. However this leaves a span of 500 years, so we have not narrowed down Diophantus's dates a great deal by these pieces of information.

There is another piece of information which was accepted for many years as giving fairly accurate dates. Heath [3] quotes from a letter by Michael Psellus who lived in the last half of the 11th century. Psellus wrote (Heath's translation in [3]):-

> *Diophantus dealt with* [*Egyptian arithmetic*] *more accurately, but the very learned Anatolius collected the most essential parts of the doctrine as stated by Diophantus in a different way and in the most succinct form, dedicating his work to Diophantus.*

Psellus also describes in this letter the fact that Diophantus gave different names to powers of the unknown to those given by the Egyptians. This letter was first published by Paul Tannery in [7] and in that work he comments that he believes that Psellus is quoting from a commentary on Diophantus which is now lost and was probably written by Hypatia. However, the quote given above has been used to date Diophantus using the theory that the Anatolius referred to here is the bishop of Laodicea who was a writer and teacher of mathematics and lived in the third century. From this it was deduced that Diophantus wrote around 250 AD and the dates we have given for him are based on this argument.

Knorr in [16] criticises this interpretation, however:-

> *But one immediately suspects something is amiss: it seems peculiar that someone would compile an abridgement of another man's work and then dedicate it to him, while the qualification "in a different way", in itself vacuous, ought to be redundant, in view of the terms "most essential" and "most succinct".*

Knorr gives a different translation of the same passage (showing how difficult the study of Greek mathematics is for anyone who is not an expert in classical Greek) which has a remarkably different meaning:-

> *Diophantus dealt with* [*Egyptian arithmetic*] *more accurately, but the very learned Anatolius, having collected the most essential parts of that man's doctrine, to a different Diophantus most succinctly addressed it.*

The conclusion of Knorr as to Diophantus's dates is [16]:-

> *... we must entertain the possibility that Diophantus lived earlier than the third century, possibly even earlier that Heron in the first century.*

The most details we have of Diophantus's life (and these may be totally fictitious) come from the Greek Anthology, compiled by Metrodorus around 500 AD. This collection of puzzles contain one about Diophantus which says:-

> *... his boyhood lasted* $^1/_6$*th of his life; he married after* $^1/_7$*th more; his beard grew after* $^1/_{12}$*th more, and his son was born* 5 *years later; the son lived to half his father's age, and the father died* 4 *years after the son.*

So he married at the age of 26 and had a son who died at the age of 42, four years before Diophantus himself died aged 84. Based on this information we have given him a life span of 84 years.

The *Arithmetica* is a collection of 130 problems giving numerical solutions of determinate equations (those with a unique solution), and indeterminate equations. The method for solving the latter is now known as Diophantine analysis. Only six of the original 13 books were thought to have survived and it was also thought that the others must have been lost quite soon after they were written. There are many Arabic translations, for example by Abu'l-Wafa, but only material from these six books appeared. Heath writes in [4] in 1920:-

> *The missing books were evidently lost at a very early date.* Paul Tannery *suggests that* Hypatia*'s commentary extended only to the first six books, and that she left untouched the remaining seven, which, partly as a consequence, were first forgotten and then lost.*

However, an Arabic manuscript in the library Astan-i Quds (The Holy Shrine library) in Meshed, Iran has a title claiming it is a translation by Qusta ibn Luqa, who died in 912, of Books IV to VII of *Arithmetica* by Diophantus of Alexandria. F Sezgin made this remarkable discovery in 1968. In [19] and [20] Rashed compares the four books in this Arabic translation with the known six Greek books and claims that this text is a translation of the lost books of Diophantus. Rozenfeld, in reviewing these two articles is, however, not completely convinced:-

> *The reviewer, familiar with the Arabic text of this manuscript, does not doubt that this manuscript is the translation from the Greek text written in Alexandria but the great difference between the Greek books of Diophantus's Arithmetic combining questions of algebra with deep questions of the theory of numbers and these books containing only algebraic material make it very probable that this text was written not by Diophantus but by some one of his commentators* (*perhaps* Hypatia ?).

It is time to take a look at this most outstanding work on algebra in Greek mathematics. The work considers the solution of many problems concerning linear and quadratic equations, but considers only positive rational solutions to these problems. Equations which would lead to solutions which are negative or irrational square roots, Diophantus considers as useless. To give one specific example, he calls the equation $4 = 4x + 20$ 'absurd' because it would lead to a meaningless answer. In other words how could a problem lead to the solution -4 books? There is no evidence to suggest that Diophantus realised that a quadratic equation could have two solutions. However, the fact that he was always satisfied with a rational solution and did not require a whole number is more sophisticated than we might realise today.

Diophantus looked at three types of quadratic equations $ax^2 + bx = c$, $ax^2 = bx + c$ and $ax^2 + c = bx$. The reason why there were three cases to Diophantus, while today we have only one case, is that he did not have any notion for zero and he avoided negative coefficients by considering the given numbers a, b, c to all be positive in each of the three cases above.

There are, however, many other types of problems considered by Diophantus. He solved problems such as pairs of simultaneous quadratic equations.
Consider $y + z = 10$, $yz = 9$. Diophantus would solve this by creating a single quadratic equation in x. Put $2x = y - z$ so, adding $y + z = 10$ and $y - z = 2x$, we have $y = 5 + x$, then subtracting them gives $z = 5 - x$. Now

$$9 = yz = (5 + x)(5 - x) = 25 - x^2, \text{ so } x^2 = 16, x = 4$$

leading to $y = 9$, $z = 1$.

In Book III, Diophantus solves problems of finding values which make two linear expressions simultaneously into squares. For example he shows how to find x to make $10x + 9$ and $5x + 4$ both squares (he finds $x = 28$). Other problems seek a value for x such that particular types of polynomials in x up to degree 6 are squares. For example he solves the problem of finding x such that $x^3 - 3x^2 + 3x + 1$ is a square in Book VI. Again in Book VI he solves problems such as finding x such that simultaneously $4x + 2$ is a cube and $2x + 1$ is a square (for which he easily finds the answer $x = 3/2$).

Another type of problem which Diophantus studies, this time in Book IV, is to find powers between given limits. For example to find a square between 5/4 and 2 he multiplies both by 64, spots the square 100 between 80 and 128, so obtaining the solution 25/16 to the original problem. In Book V he solves problems such as writing 13 as the sum of two square each greater than 6 (and he gives the solution 66049/10201 and 66564/10201). He also writes 10 as the sum of three squares each greater than 3, finding the three squares

> 1745041/505521, 1651225/505521, 1658944/505521.

Heath looks at number theory results of which Diophantus was clearly aware, yet it is unclear whether he had a proof. Of course these results may have been proved in other books written by Diophantus or he may have felt they were "obviously" true due to his experimental evidence. Among such results are [4]:-

> *... no number of the form 4n + 3 or 4n - 1 can be the sum of two squares;*
>
> *... a number of the form 24n + 7 cannot be the sum of three squares.*

Diophantus also appears to know that every number can be written as the sum of four squares. If indeed he did know this result it would be truly remarkable for even Fermat, who stated the result, failed to provide a proof of it and it was not settled until Lagrange proved it using results due to Euler.

Although Diophantus did not use sophisticated algebraic notation, he did introduce an algebraic symbolism that used an abbreviation for the unknown and for the powers of the unknown. As Vogel writes in [1]:-

> *The symbolism that Diophantus introduced for the first time, and undoubtedly devised himself, provided a short and readily comprehensible means of expressing an equation... Since an abbreviation is also employed for the word "equals", Diophantus took a fundamental step from verbal algebra towards symbolic algebra.*

One thing will be clear from the examples we have quoted and that is that Diophantus is concerned with particular problems more often than with general methods. The reason for this is that although he made important advances in symbolism, he still lacked the necessary notation to express more general methods. For instance he only had notation for one unknown and, when problems involved more than a single unknown, Diophantus was reduced to expressing "first unknown", "second unknown", etc. in words. He also lacked a symbol for a general number n. Where we would write $(12 + 6n)/(n^2 - 3)$, Diophantus has to write in words:-

> *... a sixfold number increased by twelve, which is divided by the difference by which the square of the number exceeds three.*

Despite the improved notation and that Diophantus introduced, algebra had a long way to go before really general problems could be written down and solved succinctly.

Fragments of another of Diophantus's books *On polygonal numbers*, a topic of great interest to Pythagoras and his followers, has survived. In [1] it is stated that this work contains:-

> *... little that is original,* [*and*] *is immediately differentiated from the Arithmetica by its use of geometric proofs.*

Diophantus himself refers to another work which consists of a collection of lemmas called *The Porisms* but this book is entirely lost. We do know three lemmas contained in *The Porisms* since Diophantus refers to them in the *Arithmetica*. One such lemma is that the difference of the cubes of two rational numbers is equal to the sum of the cubes of two other rational numbers, i.e. given any numbers a, b then there exist numbers c, d such that $a^3 - b^3 = c^3 + d^3$.

Another extant work *Preliminaries to the geometric elements*, which has been attributed to Heron, has been studied recently in [16] where it is suggested that the attribution to Heron is incorrect and that the work is due to Diophantus. The author of the article [14] thinks that he may have identified yet another work by Diophantus. He writes:-

> *We conjecture the existence of a lost theoretical treatise of Diophantus, entitled "Teaching of the elements of arithmetic". Our claims are based on a scholium of an anonymous Byzantine commentator.*

European mathematicians did not learn of the gems in Diophantus's *Arithmetica* until Regiomontanus wrote in 1463:-

> *No one has yet translated from the Greek into Latin the thirteen Books of Diophantus, in which the very flower of the whole of arithmetic lies hid...*

Bombelli translated much of the work in 1570 but it was never published. Bombelli did borrow many of Diophantus's problems for his own *Algebra*. The most famous Latin translation of the Diophantus's *Arithmetica* is due to Bachet in 1621 and it is that edition which Fermat studied. Certainly Fermat was inspired by this work which has become famous in recent years due to its connection with Fermat's Last Theorem.

We began this article with the remark that Diophantus is often regarded as the 'father of algebra' but there is no doubt that many of the methods for solving linear and quadratic equations go back to Babylonian mathematics. For this reason Vogel writes [1]:-

> *... Diophantus was not, as he has often been called, the father of algebra. Nevertheless, his remarkable, if unsystematic, collection of indeterminate problems is a singular achievement that was not fully appreciated and further developed until much later.*

Article by: *J J O'Connor* and *E F Robertson*

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JOC/EFR February 1999

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The URL of this page is:
http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Diophantus.html

Rafael Bombelli

Born: Jan 1526 in Bologna, Italy
Died: 1572 in (probably) Rome, Italy

Show birthplace location

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Rafael Bombelli's father was Antonio Mazzoli but he changed his name from Mazzoli to Bombelli. It is perhaps worth giving a little family background. The Bentivoglio family ruled over Bologna from 1443. Sante Bentivoglio was "signore" (meaning lord) of Bologna from 1443 and he was succeeded by Giovanni II Bentivoglio who improved the city of Bologna, in particular developing its waterways. The Mazzoli family were supporters of the Bentivoglio family but their fortunes changed when Pope Julius II took control of Bologna in 1506, driving the Bentivoglio family into exile. An attempt to regain control in 1508 was defeated and Antonio Mazzoli's grandfather, like several other supporters of the failed Bentivoglio coup, were executed. The Mazzoli family suffered for many years by having their property confiscated, but the property was returned to Antonio Mazzoli, Rafael Bombelli's father.

Antonio Mazzoli was able to return to live in Bologna. There he carried on his trade as a wool merchant and married Diamante Scudieri, a tailor's daughter. Rafael Bombelli was their eldest son, and he was one of a family of six children. Rafael received no university education. He was taught by an engineer-architect Pier Francesco Clementi so it is perhaps not too surprising that Bombelli himself should turn to that occupation. Bombelli found himself a patron in Alessandro Rufini who was a Roman noble, later to become the Bishop of Melfi.

It is unclear exactly how Bombelli learnt of the leading mathematical works of the day, but of course he lived in the right part of Italy to be involved in the major events surrounding the solution of cubic and quartic equations. Scipione del Ferro, the first to solve the cubic equation was the professor at Bologna, Bombelli's home town, but del Ferro died the year that Bombelli was born. The contest between Fior and Tartaglia (see Tartaglia's biography) took place in 1535 when Bombelli was nine years old, and Cardan's major work on the topic *Ars Magna* was published in 1545. Clearly Bombelli had studied Cardan's work and he also followed closely the very public arguments between Cardan, Ferrari and Tartaglia which culminated in the contest between Ferrari and Tartaglia in Milan in 1548 (see Ferrari's biography for details).

From about 1548 Pier Francesco Clementi, Bombelli's teacher, worked for the Apostolic Camera, a specialised department of the papacy in Rome set up to deal with legal and financial matters. The Apostolic Camera employed Clementi to reclaim marshes near Foligno on the Topino River, southeast of Perugia in central Italy. This region had became part of the Papal States in 1439. It is probable that

Bombelli assisted his teacher Clementi with this project, but we have no direct evidence that this was the case. We certainly know that around 1549 Bombelli became interested in another reclamation project in a neighbouring region.

It was in 1549 that Alessandro Rufini, Bombelli's patron, acquired the rights to reclaim that part of the marshes of the Val di Chiana which belonged to the Papal States. The Val di Chiana is a fairly central region in the Tuscan Apennines which was not well drained either by the Arno river which runs north west going through Florence and Pisa to the sea, or by the Tiber which runs south through Rome. By 1551 Bombelli was in the Val di Chiana recording the boundaries to the land that was to be reclaimed. He worked on this project until 1555 when there was an interruption to the reclamation work.

While Bombelli was waiting for the Val di Chiana project to recommence, he decided to write an algebra book. He had felt that the reason for the many arguments between leading mathematicians was the lack of a careful exposition of the subject. Only Cardan had, in Bombelli's opinion, explored the topic in depth and his great masterpiece was not accessible to people without a thorough grasp of mathematics. Bombelli felt that a self-contained text which could be read by those without a high level of mathematical training would be beneficial. He wrote in the preface of his book [2] (see also [3]):-

> *I began by reviewing the majority of those authors who have written on* [*algebra*] *up to the present, in order to be able to serve instead of them on the matter, since there are a great many of them.*

By 1557, the work at Val di Chiana still being suspended, Bombelli had begun writing his algebra text. We will study in detail the contents of the work below. Suffice to say for the moment that, in 1560 when work at Val di Chiana recommenced, Bombelli had not completed his algebra book.

Work at the Val di Chiana marshes could not have been far from completion when it had been suspended, for it was completed before the end of 1560. The scheme was a great success and through the project Bombelli gained a high reputation as an hydraulic engineer. In 1561 Bombelli went to Rome but failed in an attempt to repair the Santa Maria bridge over the Tiber. However, with reputation still high, Bombelli was taken on as a consultant for a project to drain the Pontine Marshes. These marshes in the Lazio region of south-central Italy had been an area where malaria had been a health hazard since the period of the Roman Republic. Several emperors and popes made unsuccessful attempts to reclaim the area but all, including the one which Bombelli acted as consultant on for Pope Pius IV, came to nothing. [It was not until 1928 that the Pontine Marshes were finally drained.]

On one of Bombelli's visits to Rome he made an exciting mathematical discovery. Antonio Maria Pazzi, who taught mathematics at the University of Rome, showed Bombelli a manuscript of Diophantus's *Arithmetica* and, after Bombelli had examined it, the two men decided to make a translation. Bombelli wrote in [2] (see also [3]):-

> ... [*we*], *in order to enrich the world with a work so finely made, decided to translate it and we have translated five of the books* (*there being seven in all*)*; the remainder we were not able to finish because of pressure of work on one or other.*

Despite never completing the task, Bombelli began to revise his algebra text in the light of what he had discovered in Diophantus. In particular, 143 of the 272 problems which Bombelli gives in Book III are taken from Diophantus. Bombelli does not identify which problems are his own and which are due to

Diophantus, but he does give full credit to Diophantus acknowledging that he has borrowed many of the problems given in his text from the *Arithmetica*.

Bombelli's *Algebra* was intended to be in five books. The first three were published in 1572 and at the end of the third book he wrote that [1]:-

> *... the geometrical part, Books IV and V, is not yet ready for the publisher, but its publication will follow shortly.*

Unfortunately Bombelli was never able to complete these last two volumes for he died shortly after the publication of the first three volumes. In 1923, however, Bombelli's manuscript was discovered in a library in Bologna by Bortolotti. As well as a manuscript version of the three published books, there was the unfinished manuscript of the other two books. Bortolotti published the incomplete geometrical part of Bombelli's work in 1929. Some results from Bombelli's incomplete Book IV are also described in [17] where author remarks that Bombelli's methods are related to the geometrical procedures of Omar Khayyam.

Bombelli's *Algebra* gives a thorough account of the algebra then known and includes Bombelli's important contribution to complex numbers. Before looking at his remarkable contribution to complex numbers we should remark that Bombelli first wrote down how to calculate with negative numbers. He wrote (see [2] or [3]):-

> *Plus times plus makes plus*
> *Minus times minus makes plus*
> *Plus times minus makes minus*
> *Minus times plus makes minus*
> *Plus* 8 *times plus* 8 *makes plus* 64
> *Minus* 5 *times minus* 6 *makes plus* 30
> *Minus* 4 *times plus* 5 *makes minus* 20
> *Plus* 5 *times minus* 4 *makes minus* 20

As Crossley notes in [3]:-

> *Bombelli is explicitly working with signed numbers. He has no reservations about doing this, even though in the problems he subsequently treats he neglects possible negative solutions.*

In Bombelli's *Algebra* there is even a geometric proof that minus time minus makes plus; something which causes many people difficulty even today despite our mathematical sophistication.

Bombelli, himself, did not find working with complex numbers easy at first, writing in [2] (see also [3]):-

> *And although to many this will appear an extravagant thing, because even I held this opinion some time ago, since it appeared to me more sophistic than true, nevertheless I searched hard and found the demonstration, which will be noted below. ... But let the reader apply all his strength of mind, for* [*otherwise*] *even he will find himself deceived.*

Bombelli was the first person to write down the rules for addition, subtraction and multiplication of complex numbers. He writes $+\sqrt{-n}$ as "plus of minus", $-\sqrt{-n}$ as "minus of minus", and gives rules such as (see [2] or [3]):-

> *Plus of minus times plus of minus makes minus* [$+\sqrt{-n} \cdot +\sqrt{-n} = -n$]
> *Plus of minus times minus of minus makes plus* [$+\sqrt{-n} \cdot -\sqrt{-n} = +n$]
> *Minus of minus times plus of minus makes plus* [$-\sqrt{-n} \cdot +\sqrt{-n} = +n$]
> *Minus of minus times minus of minus makes minus* [$-\sqrt{-n} \cdot -\sqrt{-n} = -n$]

After giving this description of multiplication of complex numbers, Bombelli went on to give rules for adding and subtracting them.

He then showed that, using his calculus of complex numbers, correct real solutions could be obtained from the Cardan-Tartaglia formula for the solution to a cubic even when the formula gave an expression involving the square roots of negative numbers.

Finally we should make some comments on Bombelli's notation. Although authors such as Pacioli had made limited use of notation, others such as Cardan had used no symbols at all. Bombelli, however, used quite sophisticated notation. It is worth remarking that the printed version of his book uses a slightly different notation from his manuscript, and this is not really surprising for there were problems printing mathematical notation which to some extent limited the type of notation which could be used in print.

Modern notation	Bombelli printed	Bombelli written
$5x$	$\overset{\underset{\smile}{1}}{5}$	$\overset{\underset{\smile}{1}}{5}$
$5x^2$	$\overset{\underset{\smile}{2}}{5}$	$\overset{\underset{\smile}{2}}{5}$
$\sqrt{4+\sqrt{6}}$	$\text{Rq}\lfloor 4\text{pRq}6 \rfloor$	$\text{R}\lfloor 4\text{pR}6 \rfloor$
$\sqrt[3]{2+\sqrt{0-121}}$	$\text{Rc}\lfloor 2\text{pRq}\lfloor 0\text{m}121 \rfloor \rfloor$	$\text{R}^3\lfloor 2\text{pR}\lfloor 0\text{m}121 \rfloor \rfloor$

Here are some examples of Bombelli's notation.

Despite the delay in publication, Bombelli's *Algebra* was a very influential work and led to Leibniz praising Bombelli saying he was an:-

> *... outstanding master of the analytical art.*

Jayawardene writes in [1] that in his treatment of complex numbers Bombelli:-

> *... showed himself to be far ahead of his time, for his treatment was almost that followed today.*

Crossley writes in [3]:-

> *Thus we have an engineer, Bombelli, making practical use of complex numbers perhaps*

> *because they gave him useful results, while Cardan found the square roots of negative numbers useless. Bombelli is the first to give a treatment of any complex numbers... It is remarkable how thorough he is in his presentation of the laws of calculation of complex numbers...*

It seems to be quite fair to describe Bombelli as the inventor of complex numbers. Nobody before him had given rules for working with such numbers, nor had they suggested that working with such numbers might prove useful. Dieudonné does not appear to agree with this assessment, however, for in his review of [5] and [6], he writes:-

> *... imaginaries had been used long before Bombelli's book, and it is therefore not quite justified to call him the "first discoverer" of complex numbers.*

I [EFR] feel that Dieudonné is wrong here as I believe he is when he writes that Bombelli's *Algebra*

> *... did not sell very well, nor apparently did it have much influence on later developments.*

I think that Bombelli's *Algebra* is one of the most remarkable achievements of 16th century mathematics, and he must be credited with understanding the importance of complex numbers at a time when clearly nobody else did.

Article by: *J J O'Connor* and *E F Robertson*

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